Summer Assignment Pre-Calculus

Name ______________________________

Equations in One Variable:

Example 1 Solve $100 - 8x = 140$

$$\begin{aligned} 100 - 8x &= 140 \\ 100 - 8x - 100 &= 140 - 100 \\ -8x &= 40 \\ x &= -5 \end{aligned}$$

Check $100 - 8(-5) = 140\ \surd$

Example 2 Solve $4x + 5y = 100$ for y.

$$\begin{aligned} 4x + 5y &= 100 \\ 4x + 5y - 4x &= 100 - 4x \\ 5y &= 100 - 4x \\ y &= \tfrac{1}{5}(100 - 4x) \\ y &= 20 - \tfrac{4}{5}x \end{aligned}$$

Example 3 Solve $|2x - 3| = 17$. Check you solution.

Case 1

$$\begin{aligned} 2x - 3 &= 17 \\ 2x - 3 + 3 &= 17 + 3 \\ 2x &= 20 \\ x &= 10 \end{aligned}$$

Check $|2(10) - 3| = 17\ \surd$

Case 2

$$\begin{aligned} 2x - 3 &= -17 \\ 2x - 3 + 3 &= -17 + 3 \\ 2x &= -14 \\ x &= -7 \end{aligned}$$

Check $|2(-7) - 3| = 17\ \surd$

Example 4 Solve $2\sqrt{4x+8} - 4 = 8$

$2\sqrt{4x+8} - 4 = 8$	Original equation
$2\sqrt{4x+8} = 12$	Add 4 to each side
$\sqrt{4x+8} = 6$	Isolate the radical
$4x + 8 = 36$	Square each side
$4x = 28$	Subtract 8 from each side
$x = 7$	Divide each side by 4

Check $2\sqrt{4(7)+8} - 4 = 8\ \surd$

Example 5 Solve $\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$

$\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$ Original equation

$10(x+1)\left(\frac{9}{10} + \frac{2}{x+1}\right) = 10(x+1)\left(\frac{2}{5}\right)$

Multiply all terms by the common denominator

$9(x+1) + 2(10) = 4(x+1)$ Multiply

$9x + 9 + 20 = 4x + 4$ Distributive Property

$5x = -25$

Subtract $4x$ and 29 from each side

$x = -5$ Divide each side by 5

Check $\frac{9}{10} + \frac{2}{-5+1} = \frac{2}{5}\ \surd$

Example 6 Solve each equation by factoring.

$3x^2 = 15x$	Original equation
$3x^2 - 15x = 0$	Subtract $15x$ from both sides
$3x(x - 5) = 0$	Factor the binomial
$3x = 0$ *or* $x - 5 = 0$	Zero Product Property

The solution set is $\{0, 5\}$.

$4x^2 - 5x = 21$	Original equation
$4x^2 - 5x - 21 = 0$	Subtract 21 from both sides
$(4x + 7)(x - 3) = 0$	Factor the trinomial
$4x + 7 = 0$ *or* $x - 3 = 0$	Zero Product Property
$x = -\frac{7}{4}$ *or* $x = 3$	Solve each equation

The solution set is $\left\{-\frac{7}{4}, 3\right\}$.

Complete the Square To complete the square for a quadratic expression of the form $x^2 + bx$, follow these steps.

1. Find $\frac{b}{2}$. → 2. Square $\frac{b}{2}$. → 3. Add $\left(\frac{b}{2}\right)^2$ to $x^2 + bx$.

Example 1 **Find the value of c that makes $x^2 + 22x + c$ a perfect square trinomial. Then write the trinomial as the square of a binomial.**

Step 1 $b = 22$; $\frac{b}{2} = 11$

Step 2 $11^2 = 121$

Step 3 $c = 121$

The trinomial is $x^2 + 22x + 121$, which can be written as $(x + 11)^2$.

Example 2 **Solve $2x^2 - 8x - 24 = 0$ by completing the square.**

$2x^2 - 8x - 24 = 0$	Original equation
$\frac{2x^2 - 8x - 24}{2} = \frac{0}{2}$	Divide each side by 2.
$x^2 - 4x - 12 = 0$	$x^2 - 4x - 12$ is not a perfect square.
$x^2 - 4x = 12$	Add 12 to each side.
$x^2 - 4x + 4 = 12 + 4$	Since $\left(-\frac{4}{2}\right)^2 = 4$, add 4 to each side.
$(x - 2)^2 = 16$	Factor the square
$x - 2 = \pm 4$	Square Root Property
$x = 6$ or $x = -2$	Solve each equation.

The solution set is $\{6, -2\}$.

Quadratic Formula The **Quadratic Formula** can be used to solve *any* quadratic equation once it is written in the form $ax^2 + bx + c = 0$.

Quadratic Formula	The solutions of $ax^2 + bx + c = 0$, with $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example **Solve $x^2 - 5x = 14$ by using the Quadratic Formula.**

Rewrite the equation as $x^2 - 5x - 14 = 0$.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-14)}}{2(1)}$	Replace a with 1, b with -5, and c with -14.
$= \frac{5 \pm \sqrt{81}}{2}$	Simplify
$= \frac{5 \pm 9}{2}$	
$= 7$ or -2	

The solutions are -2 and 7.

Solve each equation. Check your solution.

1. $x + 4 = 5x + 2$
2. $3x = 2x + 5$
3. $4x + 20 - 6 = 34$
4. $x - \frac{2x}{5} = 3$
5. $2.2x + 0.8x + 5 = 4x$
6. $|2x - 3| = 29$
7. $-3|4x - 9| = 24$
8. $4|2x - 7| + 5 = 9$
9. $x - 3(2x + 3) = 8 - 5x$

Solve each equation or formula for the specified variable.

10. $I = prt\ ;\ p$
11. $y = \frac{1}{4}x - 12\ ; x$
12. $A = \frac{x+y}{2}; y$
13. $A = 2\pi r^2 + 2\pi rh\ ; h$

Solve each equation. Check your solution.

14. $\frac{x}{x-1} = \frac{1}{2}$
15. $\frac{2}{x+1} = \frac{1}{x-2}$
16. $\frac{1}{x+3} + \frac{5}{x^2-9} = \frac{2}{x-3}$
17. $\frac{12x+19}{x^2+7x+12} - \frac{3}{x+3} = \frac{5}{x+4}$
18. $3 + 2x\sqrt{3} = 5$
19. $\sqrt{x^2 + 7x} = \sqrt{7x - 9}$
20. $4\sqrt[3]{2x + 11} - 2 = 10$
21. $\sqrt{9x - 11} = x + 1$
22. $\frac{4x}{9} - \frac{1}{3} = x + \frac{5}{3}$

Solve each equation by factoring.

23. $x^2 = 64$
24. $x^2 - 3x + 2 = 0$
25. $x^2 - 9x = 0$
26. $x^2 - 4x = 21$
27. $4x^2 + 5x - 6 = 0$
28. $3x^2 - 13x - 10 = 0$

Solve each equation by completing the square.

29. $x^2 - 4x - 5 = 0$
30. $2x^2 - 3x + 1 = 0$
31. $25x^2 + 40x - 9 = 0$

Solve each equation by using the Quadratic Formula.

32. $3x^2 + 5x = 2$
33. $14x^2 + 9x + 1 = 0$
34. $x^2 - \frac{3}{5}x + \frac{2}{25} = 0$

Radicals

<u>Example 1</u> Simplify $\sqrt{48}$ $\sqrt{48} = \sqrt{16}\cdot\sqrt{3} = 4\sqrt{3}$	<u>Example 3</u> Simplify $\sqrt{125x^2y^5}$ $\sqrt{125x^2y^5} = \sqrt{25}\cdot\sqrt{5}\cdot\sqrt{x^2}\cdot\sqrt{y^4}\cdot\sqrt{y} = 5\,\|x\|y^2\sqrt{5y}$
<u>Example 2</u> Simplify $\sqrt{-63}$ $\sqrt{-63} = \sqrt{-1}\cdot\sqrt{9}\cdot\sqrt{7} = 3i\sqrt{7}$	<u>Example 4</u> Simplify $\sqrt[3]{-16a^5b^7}$ $\sqrt[3]{-16a^5b^7} = \sqrt[3]{8}\cdot\sqrt[3]{2}\cdot\sqrt[3]{a^3}\cdot\sqrt[3]{a^2}\cdot\sqrt[3]{b^6}\cdot\sqrt[3]{b}$ $= -2ab^2\sqrt[3]{2a^2b}$

Simplify.

35. $\pm\sqrt{4a^{10}}$
36. $\sqrt[5]{243p^{10}}$
37. $-\sqrt[3]{m^6n^9}$
38. $\sqrt[4]{(2x)^8}$
39. $\sqrt[4]{48p^7}$
40. $\sqrt{-24a^5}$
41. $\sqrt[3]{24p^{13}q^{11}}$
42. $\sqrt[5]{-160x^8z^4}$

Relations and Functions

Example 1 Given the function $f(x) = x^2 + 2x$, find $f(3)$ and $f(5a)$.

$$f(3) = (3)^2 + 2(3) = 15 \qquad f(5a) = (5a)^2 + 2(5a) = 25a^2 + 10a$$

Arithmetic Operations

Operations with Functions	Sum	$(f+g)(x) = f(x) + g(x)$
	Difference	$(f-g)(x) = f(x) - g(x)$
	Product	$(f \cdot g)(x) = f(x) \cdot g(x)$
	Quotient	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)},\ g(x) \neq 0$

Example **Find $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for $f(x) = x^2 + 3x - 4$ and $g(x) = 3x - 2$.**

$(f+g)(x) = f(x) + g(x)$	Addition of functions
$= (x^2 + 3x - 4) + (3x - 2)$	$f(x) = x^2 + 3x - 4$, $g(x) = 3x - 2$
$= x^2 + 6x - 6$	Simplify.
$(f-g)(x) = f(x) - g(x)$	Subtraction of functions
$= (x^2 + 3x - 4) - (3x - 2)$	$f(x) = x^2 + 3x - 4$, $g(x) = 3x - 2$
$= x^2 - 2$	Simplify.
$(f \cdot g)(x) = f(x) \cdot g(x)$	Multiplication of functions
$= (x^2 + 3x - 4)(3x - 2)$	$f(x) = x^2 + 3x - 4$, $g(x) = 3x - 2$
$= x^2(3x - 2) + 3x(3x - 2) - 4(3x - 2)$	Distributive Property
$= 3x^3 - 2x^2 + 9x^2 - 6x - 12x + 8$	Distributive Property
$= 3x^3 + 7x^2 - 18x + 8$	Simplify.
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	Division of functions
$= \frac{x^2 + 3x - 4}{3x - 2},\ x \neq \frac{2}{3}$	$f(x) = x^2 + 3x - 4$ and $g(x) = 3x - 2$

Evaluate the function at each specified value of the independent variable and simplify.

43. $V = \frac{4}{3}\pi r^3;\ V(2r + 1)$

44. $q(t) = \frac{2t^2+3}{t^2};\ q(t) = 2$

45. $g(t) = 4t^2 - 3t + 5;\ g(t) - g(2)$

Find $(f+g)(x), (f-g)(x), (f \cdot g)(x)$, *and* $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

46. $f(x) = 8x - 3;\ g(x) = 4x + 5$

47. $f(x) = x^2 + x - 6;\ g(x) = x - 2$

48. $f(x) = x^2 - 1;\ g(x) = \frac{1}{x+1}$

49. $f(x) = 2x - 1;\ g(x) = 3x^2 + 11z - 4$

Linear Equations

Forms of Equations

Slope-Intercept Form of a Linear Equation	$y = mx + b$, where m is the slope and b is the y-intercept
Point-Slope Form of a Linear Equation	$y - y_1 = m(x - x_1)$, where (x_1, y_1) are the coordinates of a point on the line and m is the slope of the line

Example 1 **Write an equation in slope-intercept form for the line that has slope −2 and passes through the point (3, 7).**

Substitute for m, x, and y in the slope-intercept form.

$y = mx + b$	Slope-intercept form
$7 = (-2)(3) + b$	$(x, y) = (3, 7)$, $m = -2$
$7 = -6 + b$	Simplify
$13 = b$	Add 6 to both sides

The y-intercept is 13. The equation in slope-intercept form is $y = -2x + 13$.

Example 2 **Write an equation in slope-intercept form for the line that has slope $\frac{1}{3}$ and x-intercept 5.**

$y = mx + b$	Slope-intercept form
$0 = \left(\frac{1}{3}\right)(5) + b$	$(x, y) = (5, 0)$, $m = \frac{1}{3}$
$0 = \frac{5}{3} + b$	Simplify.
$-\frac{5}{3} = b$	Subtract $\frac{5}{3}$ from both sides.

The y-intercept is $-\frac{5}{3}$. The slope-intercept form is $y = \frac{1}{3}x - \frac{5}{3}$.

Parallel and Perpendicular Lines Use the slope-intercept or point-slope form to find equations of lines that are parallel or perpendicular to a given line. Remember that parallel lines have equal slope. The slopes of two perpendicular lines are negative reciprocals, that is, their product is −1.

Example 1 **Write an equation of the line that passes through (8, 2) and is perpendicular to the line whose equation is $y = -\frac{1}{2}x + 3$.**

The slope of the given line is $-\frac{1}{2}$. Since the slopes of perpendicular lines are negative reciprocals, the slope of the perpendicular line is 2.

Use the slope and the given point to write the equation.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 2 = 2(x - 8)$	$(x_1, y_1) = (8, 2)$, $m = 2$
$y - 2 = 2x - 16$	Distributive Prop.
$y = 2x - 14$	Add 2 to each side.

An equation of the line is $y = 2x - 14$.

Example 2 **Write an equation of the line that passes through (−1, 5) and is parallel to the graph of $y = 3x + 1$.**

The slope of the given line is 3. Since the slopes of parallel lines are equal, the slope of the parallel line is also 3.

Use the slope and the given point to write the equation.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 5 = 3(x - (-1))$	$(x_1, y_1) = (-1, 5)$, $m = 3$
$y - 5 = 3x + 3$	Distributive Prop.
$y = 3x + 8$	Add 5 to each side.

An equation of the line is $y = 3x + 8$.

50. Write an equation of the line with an undefined slope that passes through the point $(2, 1)$.

51. Write an equation of the line through the points $(-2, 1)$ and $(-4, -5)$.

52. Write an equation of the line through the point $(2.5, 6.8)$ parallel to the line $x - y = 4$.

53. Write an equation of the line through the point $\left(\frac{7}{3}, -\frac{1}{3}\right)$ perpendicular to the line that passes through the points $\left(2, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, \frac{5}{4}\right)$.

Quadratic Functions

Graph Quadratic Functions

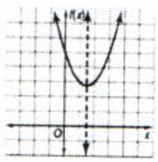
Quadratic Function	A function defined by an equation of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$
Graph of a Quadratic Function	A parabola with these characteristics: y intercept: c; axis of symmetry: $x = \frac{-b}{2a}$; x-coordinate of vertex: $\frac{-b}{2a}$

Example **Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex for the graph of $f(x) = x^2 - 3x + 5$. Use this information to graph the function.**

$a = 1$, $b = -3$, and $c = 5$, so the y-intercept is 5. The equation of the axis of symmetry is $x = \frac{-(-3)}{2(1)}$ or $\frac{3}{2}$. The x-coordinate of the vertex is $\frac{3}{2}$.

Next make a table of values for x near $\frac{3}{2}$.

x	$x^2 - 3x + 5$	$f(x)$	$(x, f(x))$
0	$0^2 - 3(0) + 5$	5	(0, 5)
1	$1^2 - 3(1) + 5$	3	(1, 3)
$\frac{3}{2}$	$\left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 5$	$\frac{11}{4}$	$\left(\frac{3}{2}, \frac{11}{4}\right)$
2	$2^2 - 3(2) + 5$	3	(2, 3)
3	$3^2 - 3(3) + 5$	5	(3, 5)



Maximum and Minimum Values

The y-coordinate of the vertex of a quadratic function is the maximum or minimum value of the function.

Maximum or Minimum Value of a Quadratic Function	The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$, opens up and has a minimum when $a > 0$. The graph opens down and has a maximum when $a < 0$.

Example **Determine whether each function has a maximum or minimum value, and find the maximum or minimum value of each function. Then state the domain and range of the function.**

a. $f(x) = 3x^2 - 6x + 7$

For this function, $a = 3$ and $b = -6$.

Since $a > 0$, the graph opens up, and the function has a minimum value.

The minimum value is the y-coordinate of the vertex. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{-6}{2(3)} = 1$.

Evaluate the function at $x = 1$ to find the minimum value.

$f(1) = 3(1)^2 - 6(1) + 7 = 4$, so the minimum value of the function is 4. The domain is all real numbers. The range is all reals greater than or equal to the minimum value, that is $\{f(x) \mid f(x) \geq 4\}$.

b. $f(x) = 100 - 2x - x^2$

For this function, $a = -1$ and $b = -2$.

Since $a < 0$, the graph opens down, and the function has a maximum value.

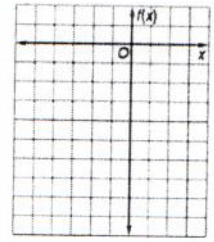
The maximum value is the y-coordinate of the vertex. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{-2}{2(-1)} = -1$.

Evaluate the function at $x = -1$ to find the maximum value.

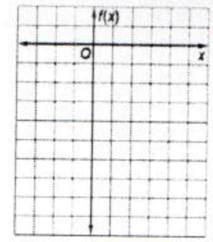
$f(-1) = 100 - 2(-1) - (-1)^2 = 101$, so the minimum value of the function is 101. The domain is all real numbers. The range is all reals less than or equal to the maximum value, that is $\{f(x) \mid f(x) \leq 101\}$.

Solve each equation by graphing.

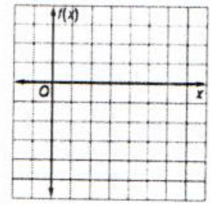
54. $x^2 + 2x - 8 = 0$



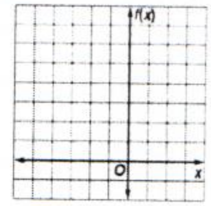
55. $x^2 - 4x - 5 = 0$



56. $x^2 - 10x + 21 = 0$

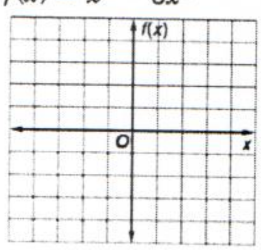


57. $x^2 + 4x + 6 = 0$

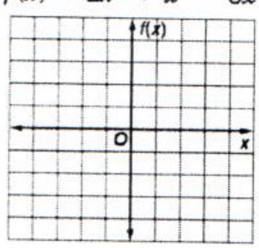


Graph each function by making a table of values. Estimate the x-coordinates at which the relative maxima and minima occur.

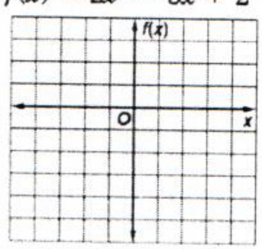
58. $f(x) = x^3 - 3x^2$



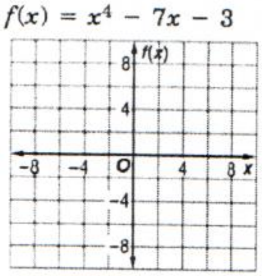
59. $f(x) = 2x^3 + x^2 - 3x$



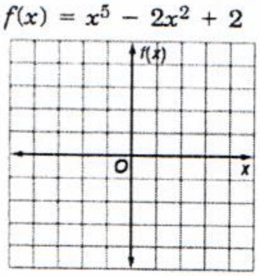
60. $f(x) = 2x^3 - 3x + 2$



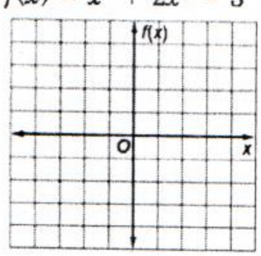
61. $f(x) = x^4 - 7x - 3$



62. $f(x) = x^5 - 2x^2 + 2$



63. $f(x) = x^3 + 2x^2 - 3$

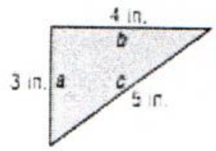


Perimeter, Circumference, and Area

The **perimeter** of a polygon is the sum of the lengths of all the sides of the polygon. The **circumference** of a circle is the distance around the circle. The **area** of a figure is the number of square units needed to cover a surface.

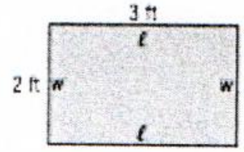
Example **Write an expression or formula for the perimeter and area of each. Find the perimeter and area. Round to the nearest tenth.**

a.



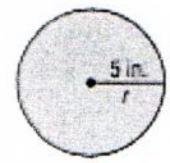
$P = a + b + c$
$= 3 + 4 + 5$
$= 12$ in.
$A = \frac{1}{2}bh$
$= \frac{1}{2}(4)(3)$
$= 6$ in^2

b.



$P = 2\ell + 2w$
$= 2(3) + 2(2)$
$= 10$ ft
$A = lw$
$= (3)(2)$
$= 6$ ft^2

c.



$C = 2\pi r$
$= 2\pi(5)$
$= 10\pi$ or about 31.4 in.
$A = \pi r^2$
$= \pi(5)^2$
$= 25\pi$ or about 78.5 in^2

Exercises

Find the perimeter or circumference and area of each figure. Round to the nearest tenth.

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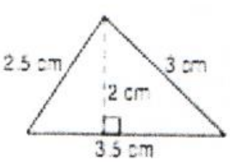

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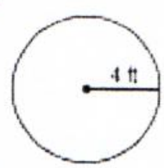

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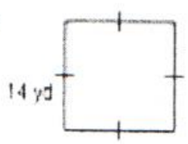

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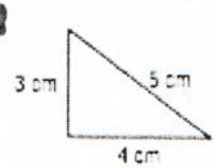
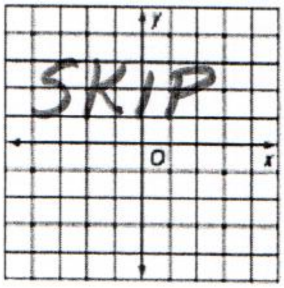
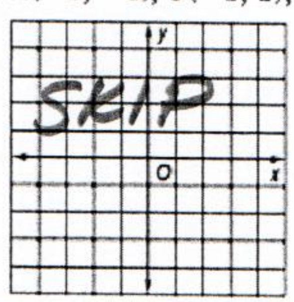


COORDINATE GEOMETRY **Graph each figure with the given vertices and identify the figure. Then find the perimeter and area of the figure.**

5. $A(-2, -4)$, $B(1, 3)$, $C(4, -4)$



6. $X(-3, -1)$, $Y(-3, 3)$, $Z(4, -1)$, $P(4, 2)$



Arc Length

An arc is part of a circle and its length is a part of the circumference of the circle.

The length of arc ℓ can be found using the following equations:

$\ell = \frac{x}{360} \cdot 2\pi r$

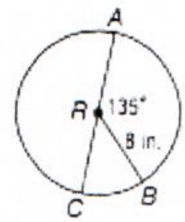
Example **Find the length of $\widehat{AB}$. Round to the nearest hundredth.**

The length of arc l, can be found using the following equation: $\widehat{AB} = \frac{x}{360} \cdot 2\pi r$

$\widehat{AB} = \frac{x}{360} \cdot 2\pi r$ Arc Length Equation

$\widehat{AB} = \frac{135}{360} \cdot 2\pi(8)$ Substitution

$\widehat{AB} \approx 18.85$ in. Use a calculator.



Exercises

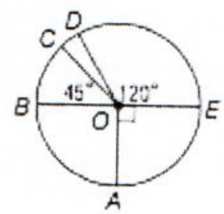
Use $\odot O$ to find the length of each arc. Round to the nearest hundredth.

61. $\widehat{DE}$ if the radius is 2 meters

62. $\widehat{DEA}$ if the diameter is 7 inches

73. $\widehat{BC}$ if $BE = 24$ feet

74. $\widehat{CBA}$ if $DO = 3$ millimeters



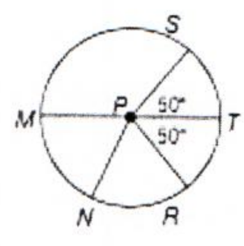
Use $\odot P$ to find the length of each arc. Round to the nearest hundredth.

5. $\widehat{RT}$, if $MT = 7$ yards SKIP

6. $\widehat{NR}$, if $PR = 12$ feet SKIP

7. $\widehat{MST}$, if $MP = 2$ inches SKIP

8. $\widehat{MRS}$, if $NS = 10$ centimeters SKIP



Areas of Sectors

A sector of a circle is a region bounded by a central angle and its intercepted arc.

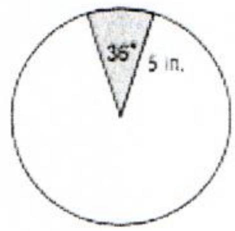
Area of a Sector	If a sector of a circle has an area of A square units, a central angle measuring $x°$, and a radius of r units, then $A = \frac{x}{360}\pi r^2$.

Example **Find the area of the shaded sector.**

$A = \frac{x}{360} \cdot \pi r^2$ Area of a sector

$= \frac{36}{360} \cdot \pi(5)^2$ $x = 36$ and $r = 5$

≈ 7.85 Use a calculator.

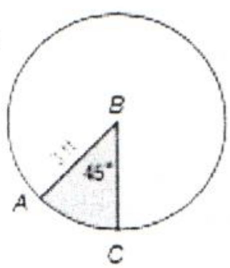


The area of the sector is about 7.85 square inches.

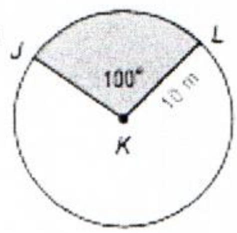
Exercises

Find the area of each shaded sector. Round to the nearest tenth.

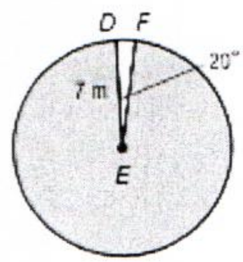
72.



73.

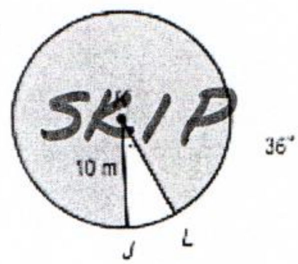


74



4.

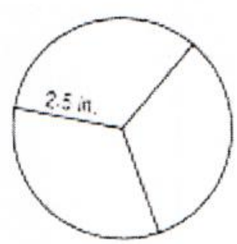
5.



6.

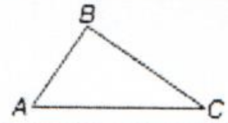
7. **SANDWICHES** For a party, Samantha wants to have finger sandwiches. She cuts sandwiches into circles. If she cuts each circle into three congruent pieces, what is the area of each piece?

SKIP

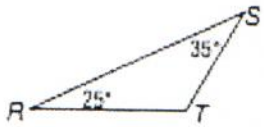


Triangle Angle-Sum Theorem

If the measures of two angles of a triangle are known, the measure of the third angle can always be found.

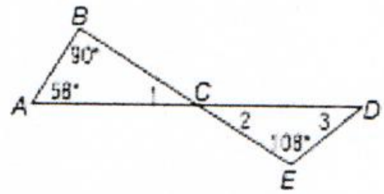
Triangle Angle Sum Theorem	The sum of the measures of the angles of a triangle is 180. In the figure at the right, $m\angle A + m\angle B + m\angle C = 180$. 

Example 1 Find $m\angle T$.



$m\angle R + m\angle S + m\angle T = 180$	Triangle Angle-Sum Theorem
$25 + 35 + m\angle T = 180$	Substitution
$60 + m\angle T = 180$	Simplify.
$m\angle T = 120$	Subtract 60 from each side.

Example 2 Find the missing angle measures.

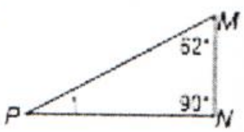


$m\angle 1 + m\angle A + m\angle B = 180$	Triangle Angle-Sum Theorem
$m\angle 1 + 58 + 90 = 180$	Substitution
$m\angle 1 + 148 = 180$	Simplify.
$m\angle 1 = 32$	Subtract 148 from each side.
$m\angle 2 = 32$	Vertical angles are congruent.
$m\angle 3 + m\angle 2 + m\angle E = 180$	Triangle Angle-Sum Theorem
$m\angle 3 + 32 + 108 = 180$	Substitution
$m\angle 3 + 140 = 180$	Simplify.
$m\angle 3 = 40$	Subtract 140 from each side.

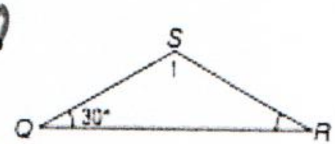
Exercises

Find the measures of each numbered angle.

15 **1.**

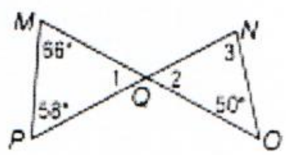


16 **2.**

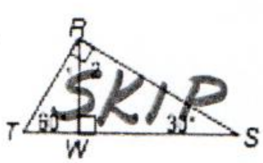


3. SKIP

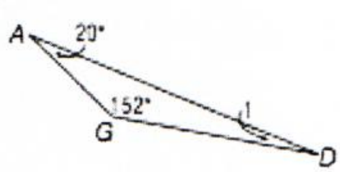
17 **4.**



5. SKIP



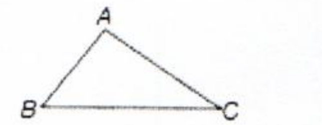
18 **6.**



Angle-Side Relationships

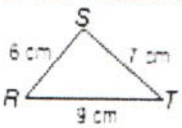
When the sides of triangles are not congruent, there is a relationship between the sides and angles of the triangles.

- If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.
- If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.



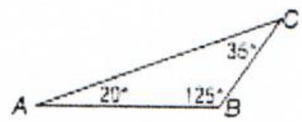
If $AC > AB$, then $m\angle B > m\angle C$.
If $m\angle A > m\angle C$, then $BC > AB$.

Example 1 **List the angles in order from smallest to largest measure.**



$\angle T, \angle R, \angle S$

Example 2 **List the sides in order from shortest to longest.**

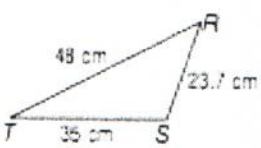


$\overline{CB}, \overline{AB}, \overline{AC}$

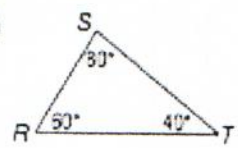
Exercises

List the angles and sides in order from smallest to largest.

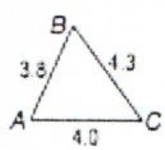
79 **1.**



80 **2.**



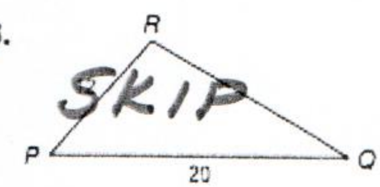
81 **3.**



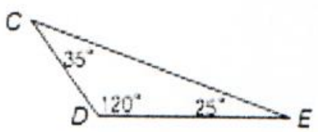
4. SKIP

5. SKIP

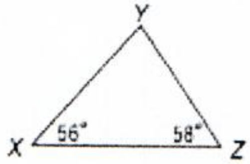
6. SKIP



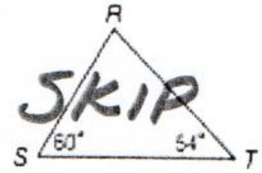
82 **7.**



83 **8.**

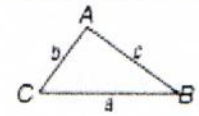


9. SKIP



The Triangle Inequality

If you take three straws of lengths 8 inches, 5 inches, and 1 inch and try to make a triangle with them, you will find that it is not possible. This illustrates the Triangle Inequality Theorem.

Triangle Inequality Theorem	The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.	 $a + b > c$ $b + c > a$ $a + c > b$

Example **The measures of two sides of a triangle are 5 and 8. Find a range for the length of the third side.**

By the Triangle Inequality Theorem, all three of the following inequalities must be true.

$5 + x > 8$	$8 + x > 5$	$5 + 8 > x$
$x > 3$	$x > -3$	$13 > x$

Therefore x must be between 3 and 13.

Exercises

Is it possible to form a triangle with the given side lengths? If not, explain why not.

84 **1.** 3, 4, 6

85 **2.** 6, 9, 15

86 **3.** 8, 8, 8

87 **4.** 2, 4, 5

88 **5.** 4, 8, 16

89 **6.** 1.5, 2.5, 3

Find the range for the measure of the third side of a triangle given the measures of two sides.

7. 1 cm and 6 cm SKIP

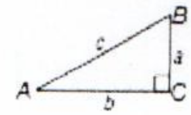
8. 12 yd and 18 yd SKIP

9. 1.5 ft and 5.5 ft SKIP

10. 82 m and 8 m SKIP

11. Suppose you have three different positive numbers arranged in order from least to greatest. What single comparison will let you see if the numbers can be the lengths of the sides of a triangle? SKIP

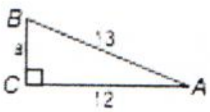
The Pythagorean Theorem In a right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse. If the three whole numbers a, b, and c satisfy the equation $a^2 + b^2 = c^2$, then the numbers a, b, and c form a **Pythagorean triple.**



$\triangle ABC$ is a right triangle, so $a^2 + b^2 = c^2$.

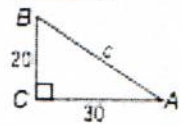
Example

a. Find a.



$a^2 + b^2 = c^2$	Pythagorean Theorem
$a^2 + 12^2 = 13^2$	$b = 12$, $c = 13$
$a^2 + 144 = 169$	Simplify.
$a^2 = 25$	Subtract.
$a = 5$	Take the positive square root of each side.

b. Find c.

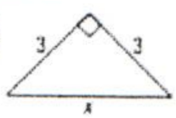


$a^2 + b^2 = c^2$	Pythagorean Theorem
$20^2 + 30^2 = c^2$	$a = 20$, $b = 30$
$400 + 900 = c^2$	Simplify.
$1300 = c^2$	Add.
$\sqrt{1300} = c$	Take the positive square root of each side.
$36.1 \approx c$	Use a calculator.

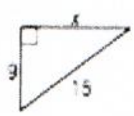
Exercises

Find x.

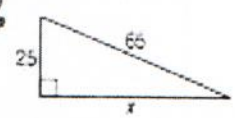
90 1.



91 2.

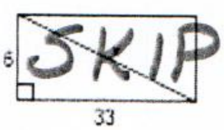


92 3.



4. SKIP

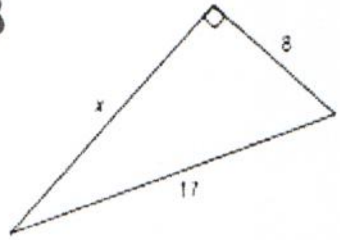
5. SKIP



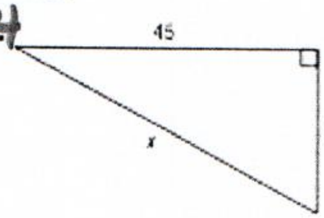
6. SKIP

Use a Pythagorean Triple to find x.

93 7.



94 8.

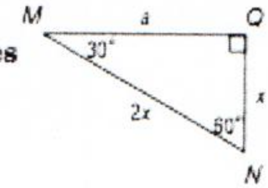


9. SKIP

Properties of 30°-60°-90° Triangles

The sides of a 30°-60°-90° right triangle also have a special relationship.

Example 1 **In a 30°-60°-90° right triangle, show that the hypotenuse is twice the shorter leg and the longer leg is $\sqrt{3}$ times the shorter leg.**



$\triangle MNQ$ is a 30°-60°-90° right triangle, and the length of the hypotenuse $\overline{MN}$ is two times the length of the shorter side $\overline{NQ}$. Using the Pythagorean Theorem,

$$\begin{aligned} a^2 &= (2x)^2 - x^2 \\ &= 4x^2 - x^2 \\ &= 3x^2 \\ a &= \sqrt{3x^2} \\ &= x\sqrt{3} \end{aligned}$$

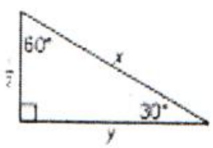
Example 2 **In a 30°-60°-90° right triangle, the hypotenuse is 5 centimeters. Find the lengths of the other two sides of the triangle.**

If the hypotenuse of a 30°-60°-90° right triangle is 5 centimeters, then the length of the shorter leg is one-half of 5, or 2.5 centimeters. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg, or $(2.5)(\sqrt{3})$ centimeters.

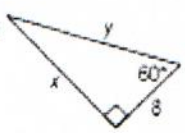
Exercises

Find x and y.

95.



96.



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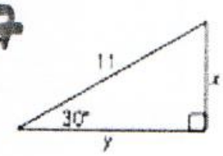

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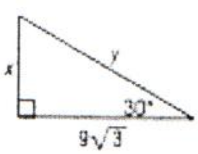

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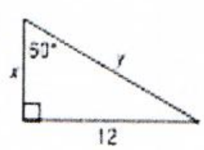

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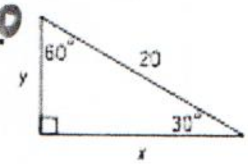


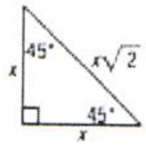
7. An equilateral triangle has an altitude length of 36 feet. Determine the length of a side of the triangle. SKIP

8. Find the length of the side of an equilateral triangle that has an altitude length of 45 centimeters. SKIP

Properties of 45°-45°-90° Triangles

The sides of a 45°-45°-90° right triangle have a special relationship.

Example 1 **If the leg of a 45°-45°-90° right triangle is x units, show that the hypotenuse is $x\sqrt{2}$ units.**



Using the Pythagorean Theorem with $a = b = x$, then

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= x^2 + x^2 \\ &= 2x^2 \\ c &= \sqrt{2x^2} \\ &= x\sqrt{2} \end{aligned}$$

Example 2 **In a 45°-45°-90° right triangle the hypotenuse is $\sqrt{2}$ times the leg. If the hypotenuse is 6 units, find the length of each leg.**

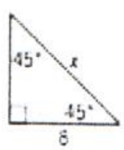
The hypotenuse is $\sqrt{2}$ times the leg, so divide the length of the hypotenuse by $\sqrt{2}$.

$$\begin{aligned} a &= \frac{6}{\sqrt{2}} \\ &= \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{6\sqrt{2}}{2} \\ &= 3\sqrt{2} \text{ units} \end{aligned}$$

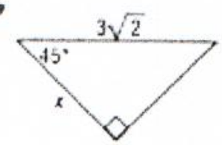
Exercises

Find x.

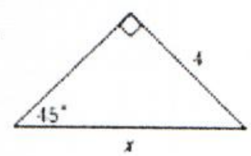
101 **1.**



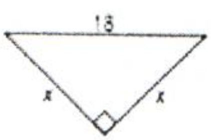
102 **2.**



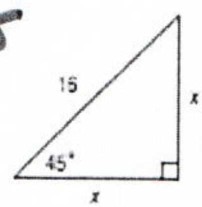
103 **3.**



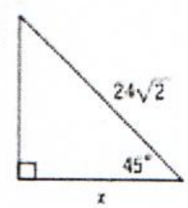
104 **4.**



105 **5.**



106 **6.**



7. If a 45°-45°-90° triangle has a hypotenuse length of 12, find the leg length. SKIP

8. Determine the length of the leg of 45°-45°-90° triangle with a hypotenuse length of 25 inches. SKIP

9. Find the length of the hypotenuse of a 45°-45°-90° triangle with a leg length of 14 centimeters. SKIP

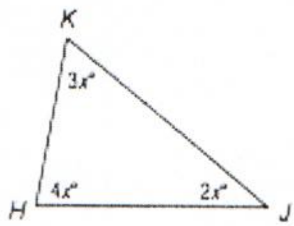
Write and Use Ratios

Write and Use Ratios A **ratio** is a comparison of two quantities by divisions. The ratio a to b, where b is not zero, can be written as $\frac{a}{b}$ or $a:b$.

Example 1 **In 2007 the Boston RedSox baseball team won 96 games out of 162 games played. Write a ratio for the number of games won to the total number of games played.**

To find the ratio, divide the number of games won by the total number of games played. The result is $\frac{96}{162}$, which is about 0.59. The Boston RedSox won about 59% of their games in 2007.

Example 2 **The ratio of the measures of the angles in $\triangle JHK$ is 2:3:4. Find the measures of the angles.**



The extended ratio 2:3:4 can be rewritten $2x{:}3x{:}4x$.

Sketch and label the angle measures of the triangle. Then write and solve an equation to find the value of x.

$2x + 3x + 4x = 180$	Triangle Sum Theorem
$9x = 180$	Combine like terms.
$x = 20$	Divide each side by 9.

The measures of the angles are 2(20) or 40, 3(20) or 60, and 4(20) or 80.

Exercises

107 1. In the 2007 Major League Baseball season, Alex Rodriguez hit 54 home runs and was at bat 583 times. What is the ratio of home runs to the number of times he was at bat?

108 2. There are 182 girls in the sophomore class of 305 students. What is the ratio of girls to total students?

109 3. The length of a rectangle is 8 inches and its width is 5 inches. What is the ratio of length to width?

110 4. The ratio of the sides of a triangle is 8:15:17. Its perimeter is 480 inches. Find the length of each side of the triangle.

111 5. The ratio of the measures of the three angles of a triangle is 7:9:20. Find the measure of each angle of the triangle.

Use Properties of Proportions

A statement that two ratios are equal is called a **proportion**. In the proportion $\frac{a}{b} = \frac{c}{d}$, where b and d are not zero, the values a and d are the **extremes** and the values b and c are the **means**. In a proportion, the product of the means is equal to the product of the extremes, so $ad = bc$. This is the Cross Product Property.

$$\frac{a}{b} = \frac{c}{d}$$

$$\underset{\text{extremes}}{\underset{\uparrow}{a \cdot d}} = \underset{\text{means}}{\underset{\uparrow}{b \cdot c}}$$

Example 1 **Solve $\frac{9}{16} = \frac{27}{x}$.**

$\frac{9}{16} = \frac{27}{x}$	
$9 \cdot x = 16 \cdot 27$	Cross Products Property
$9x = 432$	Multiply.
$x = 48$	Divide each side by 9.

Example 2 POLITICS **Mayor Hernandez conducted a random survey of 200 voters and found that 135 approve of the job she is doing. If there are 48,000 voters in Mayor Hernandez's town, predict the total number of voters who approve of the job she is doing.**

Write and solve a proportion that compares the number of registered voters and the number of registered voters who approve of the job the mayor is doing.

$$\frac{135}{200} = \frac{x}{48{,}000} \quad \begin{matrix} \leftarrow \text{voters who approve} \\ \leftarrow \text{all voters} \end{matrix}$$

$135 \cdot 48{,}000 = 200 \cdot x$	Cross Products Property
$6{,}480{,}000 = 200x$	Simplify.
$32{,}400 = x$	Divide each side by 200.

Based on the survey, about 32,400 registered voters approve of the job the mayor is doing.

Exercises

Solve each proportion.

112 1. $\frac{1}{2} = \frac{28}{x}$

SKIP 2. $\frac{3}{8} = \frac{y}{24}$

113 3. $\frac{x + 22}{x + 2} = \frac{30}{10}$

4. $\frac{3}{15} = \frac{9}{y}$ SKIP

114 5. $\frac{2x + 3}{8} = \frac{5}{4}$

6. $\frac{x + 1}{x} = \frac{3}{4}$ SKIP

7. If 3 DVDs cost $44.85, find the cost of one DVD. SKIP

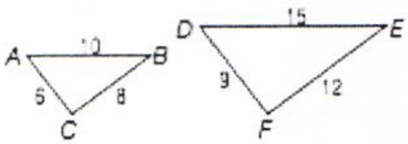
8. **BOTANY** Bryon is measuring plants in a field for a science project. Of the first 25 plants he measures, 15 of them are smaller than a foot in height. If there are 4000 plants in the field, predict the total number of plants smaller than a foot in height. SKIP

Identify Similar Triangles

Here are three ways to show that two triangles are similar.

AA Similarity	Two angles of one triangle are congruent to two angles of another triangle.
SSS Similarity	The measures of the corresponding side lengths of two triangles are proportional.
SAS Similarity	The measures of two side lengths of one triangle are proportional to the measures of two corresponding side lengths of another triangle, and the included angles are congruent.

Example 1 **Determine whether the triangles are similar.**



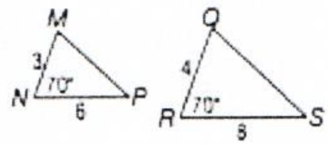
$\frac{AC}{DF} = \frac{6}{9} = \frac{2}{3}$

$\frac{BC}{EF} = \frac{8}{12} = \frac{2}{3}$

$\frac{AB}{DE} = \frac{10}{15} = \frac{2}{3}$

$\triangle ABC \sim \triangle DEF$ by SSS Similarity.

Example 2 **Determine whether the triangles are similar.**



$\frac{3}{4} = \frac{6}{8}$, so $\frac{MN}{QR} = \frac{NP}{RS}$.

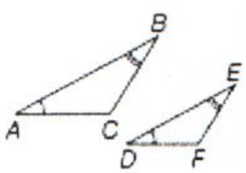
$m\angle N = m\angle R$, so $\angle N \cong \angle R$.

$\triangle NMP \sim \triangle RQS$ by SAS Similarity.

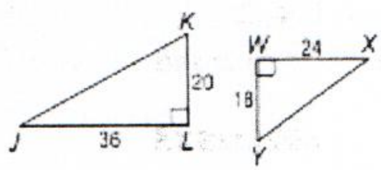
Exercises

Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning.

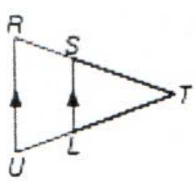
115 **1.**



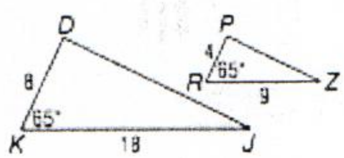
116 **2.**



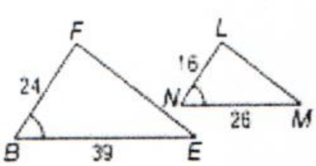
117 **3.**



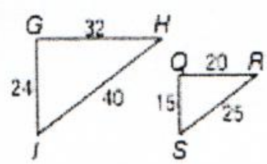
118 **4.**



119 **5.**



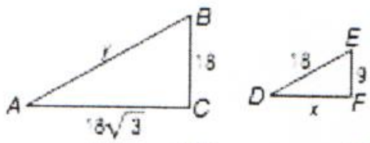
120 **6.**



Use Similar Triangles

Similar triangles can be used to find measurements.

Example 1 $\triangle ABC \sim \triangle DEF$. **Find the values of x and y.**



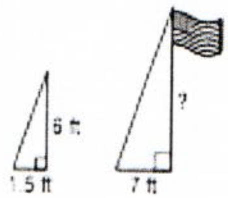
$$\frac{AC}{DF} = \frac{BC}{EF} \qquad \frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{18\sqrt{3}}{x} = \frac{18}{9} \qquad \frac{y}{18} = \frac{18}{9}$$

$$18x = 9(18\sqrt{3}) \qquad 9y = 324$$

$$x = 9\sqrt{3} \qquad y = 36$$

Example 2 **A person 6 feet tall casts a 1.5-foot-long shadow at the same time that a flagpole casts a 7-foot-long shadow. How tall is the flagpole?**

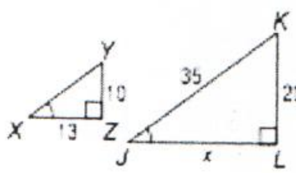


The sun's rays form similar triangles.
Using x for the height of the pole, $\frac{6}{x} = \frac{1.5}{7}$, so $1.5x = 42$ and $x = 28$.
The flagpole is 28 feet tall.

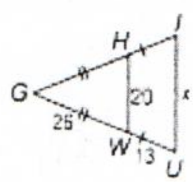
Exercises

ALGEBRA Identify the similar triangles. Then find each measure.

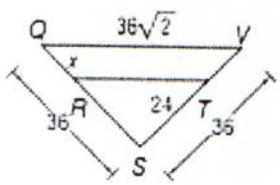
121 1. JL



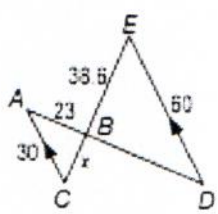
122 2. IU



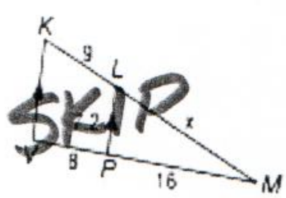
123 3. QR



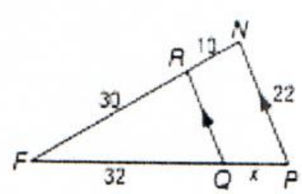
124 4. BC



5. LM

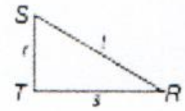


125 6. QP



127 7. The heights of two vertical posts are 2 meters and 0.45 meter. When the shorter post casts a shadow that is 0.85 meter long, what is the length of the longer post's shadow to the nearest hundredth?

Trigonometric Ratios The ratio of the lengths of two sides of a right triangle is called a **trigonometric ratio.** The three most common ratios are **sine, cosine,** and **tangent,** which are abbreviated *sin*, *cos*, and *tan*, respectively.

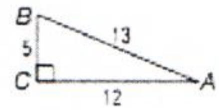


$\sin R = \dfrac{\text{leg opposite } \angle R}{\text{hypotenuse}} = \dfrac{r}{t}$

$\cos R = \dfrac{\text{leg adjacent to } \angle R}{\text{hypotenuse}} = \dfrac{s}{t}$

$\tan R = \dfrac{\text{leg opposite } \angle R}{\text{leg adjacent to } \angle R} = \dfrac{r}{s}$

Example **Find sin *A*, cos *A*, and tan *A*. Express each ratio as a fraction and a decimal to the nearest hundredth.**



$\sin A = \dfrac{\text{opposite leg}}{\text{hypotenuse}} = \dfrac{BC}{BA} = \dfrac{5}{13} \approx 0.39$

$\cos A = \dfrac{\text{adjacent leg}}{\text{hypotenuse}} = \dfrac{AC}{AB} = \dfrac{12}{13} \approx 0.92$

$\tan A = \dfrac{\text{opposite leg}}{\text{adjacent leg}} = \dfrac{BC}{AC} = \dfrac{5}{12} \approx 0.42$

Exercises

Find sin *J*, cos *J*, tan *J*, sin *L*, cos *L*, and tan *L*. Express each ratio as a fraction and as a decimal to the nearest hundredth.

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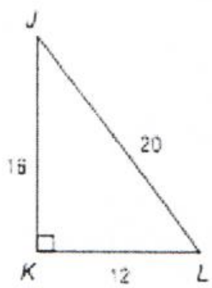

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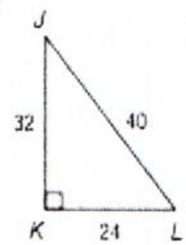

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